

## Exercise 7B

$$1 \quad \mathbf{a} \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\mathbf{b} \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \text{ so } A' = (1, -3)$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \end{pmatrix} \text{ so } B' = (3, -3)$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \text{ so } C' = (3, -2)$$

$$2 \quad \mathbf{a} \quad \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

$$\mathbf{b} \quad \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \text{ so } P' = (-1, -1)$$

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} \text{ so } Q' = (-3, -1)$$

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix} \text{ so } R' = (-3, -2)$$

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \end{pmatrix} \text{ so } S' = (-1, -2)$$

$$3 \quad \mathbf{a} \quad \begin{pmatrix} \cos 90^\circ & -\sin 90^\circ \\ \sin 90^\circ & \cos 90^\circ \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\mathbf{b} \quad \begin{pmatrix} \cos 270^\circ & -\sin 270^\circ \\ \sin 270^\circ & \cos 270^\circ \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\mathbf{c} \quad \begin{pmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{pmatrix}$$

$$\mathbf{d} \quad \begin{pmatrix} \cos 210^\circ & -\sin 210^\circ \\ \sin 210^\circ & \cos 210^\circ \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} -\sqrt{3} & 1 \\ -1 & -\sqrt{3} \end{pmatrix}$$

$$\mathbf{e} \quad \begin{pmatrix} \cos 225^\circ & -\sin 225^\circ \\ \sin 225^\circ & \cos 225^\circ \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} -\sqrt{2} & \sqrt{2} \\ -\sqrt{2} & -\sqrt{2} \end{pmatrix}$$

$$4 \quad \mathbf{a} \quad \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

So  $A' = (-1, 1)$ ,  $B' = (-1, 4)$ ,  $C' = (-2, 4)$

4 b

$$\begin{pmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{3}+1}{2} \\ \frac{1-\sqrt{3}}{2} \end{pmatrix}$$

$$\begin{pmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{4\sqrt{3}+1}{2} \\ \frac{4-\sqrt{3}}{2} \end{pmatrix}$$

$$\begin{pmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -\frac{4\sqrt{3}+2}{2} \\ \frac{4-2\sqrt{3}}{2} \end{pmatrix}$$

$$\text{So } A' = \left( -\frac{\sqrt{3}}{2} - \frac{1}{2}, \frac{1}{2} - \frac{\sqrt{3}}{2} \right),$$

$$B' = \left( -2\sqrt{3} - \frac{1}{2}, 2 - \frac{\sqrt{3}}{2} \right),$$

$$C' = (-2\sqrt{3} - 1, 2 - \sqrt{3})$$

5 a

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$\text{So } P' = (2, -2),$$

$$Q' = (3, -2), R' = (3, -4),$$

$$S' = (2, -4)$$

5 b

$$\begin{pmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ -2\sqrt{2} \end{pmatrix}$$

$$\begin{pmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} \\ -\frac{5\sqrt{2}}{2} \end{pmatrix}$$

$$\begin{pmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ -\frac{7\sqrt{2}}{2} \end{pmatrix}$$

$$\begin{pmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -\sqrt{2} \\ -3\sqrt{2} \end{pmatrix}$$

$$\text{So } P' = (0, -2\sqrt{2}),$$

$$Q' = \left( \frac{\sqrt{2}}{2}, -\frac{5\sqrt{2}}{2} \right), R' = \left( -\frac{\sqrt{2}}{2}, -\frac{7\sqrt{2}}{2} \right),$$

$$S' = (-\sqrt{2}, -3\sqrt{2})$$

6 a A represents a reflection in the  $x$ -axis.  
B represents a rotation through  $270^\circ$  anticlockwise about  $(0, 0)$ .

b  $(3, -2)$ 

$$\text{c } b = a - 3b \Rightarrow a - 4b = 0$$

$$-a = 2a - 2b \Rightarrow 3a - 2b = 0$$

$$\Rightarrow a = 0, b = 0$$

7 a Rotation through  $225^\circ$  anticlockwise about  $(0, 0)$

$$\text{b } -\frac{1}{\sqrt{2}}p + \frac{1}{\sqrt{2}}q = -\sqrt{2}$$

$$\Rightarrow -p + q = -2 \Rightarrow p - q = 2$$

$$-\frac{1}{\sqrt{2}}p = \frac{1}{\sqrt{2}}q = -2\sqrt{2}$$

$$\Rightarrow -p - q = -4 \Rightarrow p + q = 4$$

$$\Rightarrow p = 3, q = 1$$

$$7 \text{ c } \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$d \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} -\sqrt{2} \\ -2\sqrt{2} \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$$

So  $C' = (-3, -1)$

Rotation through  $45^\circ$  clockwise  
about  $(0,0)$

- 8 a Reflection in the line  $y = x$   
b Lines of the form  $y = -x + k$  for any value of  $k$

$$c \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- 9 a  $a = -0.5$   
b  $\cos \theta = -0.5 \Rightarrow \theta = 120^\circ$  or  $240^\circ$   
So possible matrices are

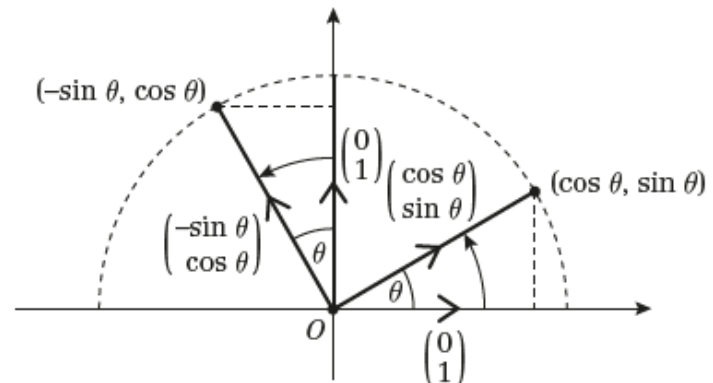
$$\begin{pmatrix} \cos 120^\circ & -\sin 120^\circ \\ \sin 120^\circ & \cos 120^\circ \end{pmatrix} = \begin{pmatrix} -0.5 & -0.866 \\ 0.866 & -0.5 \end{pmatrix}$$

$$\begin{pmatrix} \cos 240^\circ & -\sin 240^\circ \\ \sin 240^\circ & \cos 240^\circ \end{pmatrix} = \begin{pmatrix} -0.5 & 0.866 \\ -0.866 & -0.5 \end{pmatrix}$$

$$10 \text{ a } \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

- b  $a - 5b = -b \Rightarrow a = 4b$   
 $3b = a \Rightarrow a = 3b$   
So  $a = 0, b = 0$

### Challenge



The diagram shows that rotating  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$  by  $\theta$  takes it to  $\begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$  and rotating  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$  by  $\theta$  takes it to  $\begin{pmatrix} -\sin \theta \\ \cos \theta \end{pmatrix}$ . We know that the image of  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$  is the first column of the matrix representing the transformation and that the image of  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$  is the second column of the matrix representing the transformation. So the matrix representing the transformation is  $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$