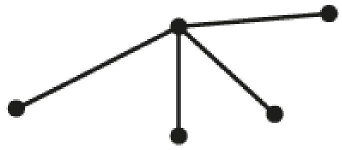
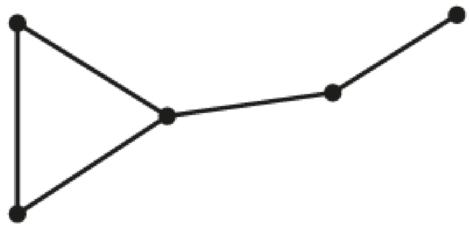
Graphs and networks 2B

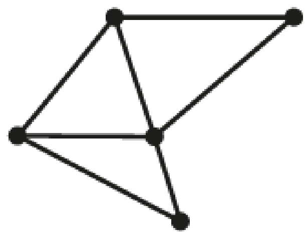
1 **a** For example;



b



c



2 **a** is not simple. There are two edges connecting **C** with **D**.

b and **c** are simple.

d is not simple. There is a loop attached to **U**.

3 **a** and **c** are connected.

b is not connected, there is no path from **J** to **G** for example.

d is not connected, there is no path from **W** to **Z** for example.

4 **a** Any four of these:

F A B D	F E D
F A C B D	F E C B D
F A B C E D	F E C A B D
F A C E D	

b Here are examples. (These all start at F, but you could start at any point.) There are others.

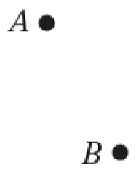
F A B D E F
F E D B A F
F A C B D E F
F E D B C A F

4 c

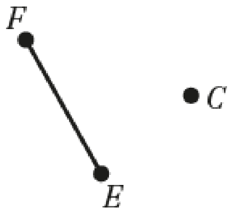
Vertex	A	B	C	D	E	F
Degree	3	3	3	2	3	2

d Here are examples. (There are many others.)

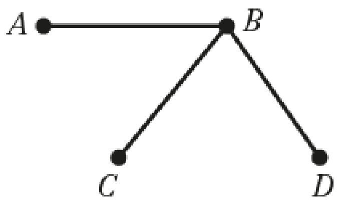
i



ii



iii



e Sum of degrees (valencies) $= 3+3+3+2+3+2=16$

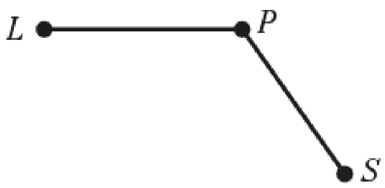
Number of edges $= 8$
Hence, indeed: sum of degrees = $2\times$ number of edges.

5

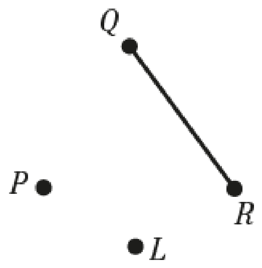
Vertex	**J**	**K**	**L**	**M**	**N**	**P**	**Q**	**R**	**S**
Degree	1	2	3	1	1	4	2	1	1

Here are some possible subgraphs (there are many others).

i



ii



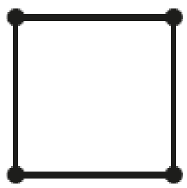
iii

Sum of degrees $= 1+2+3+1+1+4+2+1+1 = 16$

Number of edges $= 8$

Sum of degrees $= 2 \times$ number of edges for this graph

6 **a** For example,



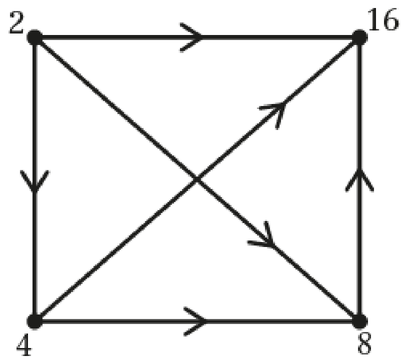
4 vertices of degree 2

b For example,

3 vertices all even, all of degree 2

c The sum of degrees $= 2 \times$ number of edges, so the sum of degrees must be even.
Any vertices of odd degree must therefore 'pair up'. So there must be an even number of vertices of odd degree.

7

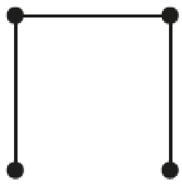


8 **a** Hamiltonian cycle is a cycle that includes every vertex.

b *PQRSTP*, *PQSRTP*, *PTSRQP*, *PTRSQP*

c A subgraph of a graph *G* is a graph made up from some of the edges and vertices of **G**.

d E.g.


or