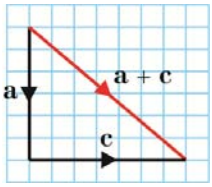
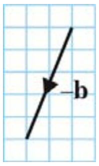
Vectors 11A

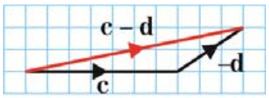
1 a



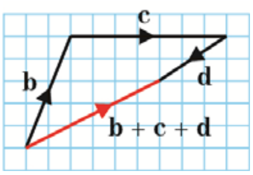
b



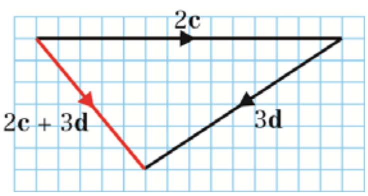
c



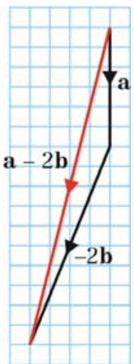
d



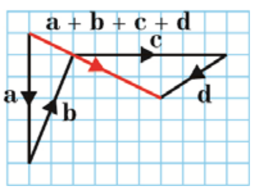
e



f



g



2 In this exercise there will usually be several correct routes to the answers because the addition law for vectors allows several options for equivalent vectors. You might reach the correct answers by a different routes to those used in these solutions.

a $\overrightarrow{AC} = \overrightarrow{AB} = 2\mathbf{b}$

b $\overrightarrow{BE} = \overrightarrow{AD}$ (parallel and equal in length)
$= \mathbf{d}$

c $\overrightarrow{HG} = \overrightarrow{BC} =$ (parallel and equal in length)
$= \overrightarrow{AB}$ (B is midpoint of $\overrightarrow{AC}$)
$= \mathbf{b}$

d $\overrightarrow{DF} = \overrightarrow{AC} =$ (parallel and equal in length)
$= 2\mathbf{b}$

e $\overrightarrow{AE} = \overrightarrow{AD} + \overrightarrow{DE}$ (triangle law of addition)
$= \overrightarrow{AD} + \overrightarrow{AB}$
($\overrightarrow{DE}$ and $\overrightarrow{AB}$ parallel and equal in length)
$= \mathbf{d} + \mathbf{b}$

f $\overrightarrow{DI} = \overrightarrow{DH} + \overrightarrow{IH}$ (triangle law of addition)
$= \overrightarrow{AD} + \overrightarrow{AB}$
($\overrightarrow{AD} = \overrightarrow{DI}$ because D is the midpoint of $\overrightarrow{AI}$, and $\overrightarrow{AB}$ is parallel and equal to $\overrightarrow{IH}$)
$= \mathbf{d} + \mathbf{b}$

g $\overrightarrow{HB} = -\overrightarrow{BH} =$ (same length, opposite direction)
$= -\overrightarrow{AI}$ (parallel and equal in length)
$= -2\mathbf{d}$

h $\overrightarrow{FE} = -\overrightarrow{EF}$ (same length, opposite direction)
$= -\overrightarrow{HG}$ (parallel and equal in length)
$= -\mathbf{b}$ (from part c)

i $\overrightarrow{AI} = \overrightarrow{AH} + \overrightarrow{IH}$ (triangle law of addition)
$= 2\mathbf{d} + \mathbf{b}$

j $\overrightarrow{BI} = \overrightarrow{BA} + \overrightarrow{AI}$

$= -\overrightarrow{AB} + \overrightarrow{AI}$
$= -\mathbf{b} + 2\mathbf{d}$

2 k $\overrightarrow{EI} = \overrightarrow{EB} + \overrightarrow{BA} + \overrightarrow{AI}$
$= -\overrightarrow{BE} - \overrightarrow{AB} - \overrightarrow{AI}$
$= -\mathbf{d} - \mathbf{b} + 2\mathbf{d}$
$= -\mathbf{b} + \mathbf{d}$

l $\overrightarrow{FB} = \overrightarrow{FD} + \overrightarrow{DA} + \overrightarrow{AB}$
$= -\overrightarrow{DF} - \overrightarrow{AD} + \overrightarrow{AB}$
$= -2\mathbf{b} - \mathbf{d} + \mathbf{b}$
$= -\mathbf{b} - \mathbf{d}$

3 a $\overrightarrow{OA} = 2\overrightarrow{OM}$ (M is the midpoint of $\overrightarrow{OA}$)
$= 2\mathbf{m}$

b $\overrightarrow{OB} = 2\overrightarrow{OP}$ (P is the mid point of $\overrightarrow{OB}$)
$= 2\mathbf{p}$

c $\overrightarrow{BN} = \frac{1}{2}\overrightarrow{BC} = \frac{1}{2}\overrightarrow{OA}$ (opposite sides parallel and equal)
$= \mathbf{m}$

d $\overrightarrow{DQ} = \overrightarrow{PD}$ ($\overrightarrow{MN}$ and $\overrightarrow{PQ}$ bisect each other)
$= \overrightarrow{OM}$ (line segments parallel and equal in length)
$= \mathbf{m}$

e $\overrightarrow{OD} = \overrightarrow{OP} + \overrightarrow{OP}$ (addition of vectors)
$= \overrightarrow{OP} + \overrightarrow{OM}$ ($\overrightarrow{PD}$ and $\overrightarrow{OM}$ are parallel and equal in length)
$= \mathbf{p} + \mathbf{m}$

f $\overrightarrow{MQ} = \overrightarrow{MO} + \overrightarrow{OP} + \overrightarrow{PQ}$ (vector addition)
$= -\overrightarrow{OM} + \overrightarrow{OP} + \overrightarrow{OA}$ ($\overrightarrow{PQ}$ and $\overrightarrow{OA}$ are parallel and equal in length)
$= -\mathbf{m} + \mathbf{p} + 2\mathbf{m}$
$= \mathbf{p} + \mathbf{m}$

g $\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$
$= \mathbf{p} + 2\mathbf{m}$

h $\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD}$ (vector addition)
$= -\overrightarrow{OA} + \overrightarrow{OD}$
$= -2\mathbf{m} + (\mathbf{p} + \mathbf{m})$
$= \mathbf{p} - \mathbf{m}$

3 i $\overrightarrow{CD} = \overrightarrow{CN} + \overrightarrow{ND}$ (vector addition)
$= \overrightarrow{MO} + \overrightarrow{PO}$ (line segments parallel and equal in length)
$= -\overrightarrow{OM} - \overrightarrow{OP}$
$= -\mathbf{m} - \mathbf{p}$

j $\overrightarrow{AP} = \overrightarrow{AO} + \overrightarrow{OP}$ (vector addition)
$= -\overrightarrow{OA} + \overrightarrow{OP}$
$= -2\mathbf{m} + \mathbf{p}$

k $\overrightarrow{BM} = \overrightarrow{BO} + \overrightarrow{OM}$ (vector addition)
$= -\overrightarrow{OB} + \overrightarrow{OM}$
$= -2\mathbf{p} + \mathbf{m}$

l $\overrightarrow{NO} = \overrightarrow{NB} + \overrightarrow{BO}$ (vector addition)
$= \overrightarrow{MO} + \overrightarrow{BO}$ ($\overrightarrow{MO}$ and $\overrightarrow{NB}$ are parallel and equal in length)
$= -\overrightarrow{OM} - \overrightarrow{OB}$
$= -\mathbf{m} - 2\mathbf{p}$

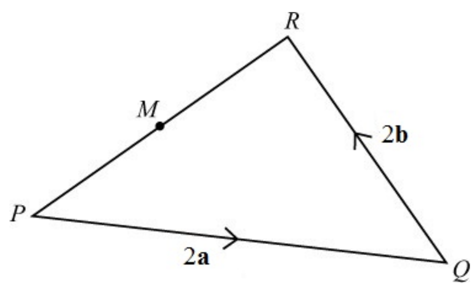
4 a $\overrightarrow{QT} = \overrightarrow{QP} + \overrightarrow{PT}$
$= -\mathbf{a} + \mathbf{d}$
$= \mathbf{d} - \mathbf{a}$

b $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QS} + \overrightarrow{SR}$
$= \mathbf{a} + \mathbf{b} + \mathbf{c}$

c $\overrightarrow{TS} = \overrightarrow{TP} + \overrightarrow{PQ} + \overrightarrow{QS}$
$= -\mathbf{d} + \mathbf{a} + \mathbf{b}$
$= \mathbf{a} + \mathbf{b} - \mathbf{d}$

d $\overrightarrow{TR} = \overrightarrow{TP} + \overrightarrow{PR}$
$= -\mathbf{d} + (\mathbf{a} + \mathbf{b} + \mathbf{c})$
$= \mathbf{a} + \mathbf{b} + \mathbf{c} - \mathbf{d}$

5



a $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$
$= 2\mathbf{a} + 2\mathbf{b}$

5 b $\overrightarrow{PM} = \frac{1}{2}\overrightarrow{PR}$

$= \frac{1}{2}(2\mathbf{a} + 2\mathbf{b})$

$= \mathbf{a} + \mathbf{b}$

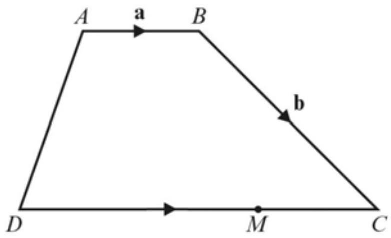
c $\overrightarrow{QM} = \overrightarrow{QP} + \overrightarrow{PM}$

$= -2\mathbf{a} + \mathbf{a} + \mathbf{b}$

$= -\mathbf{a} + \mathbf{b}$

$= \mathbf{b} - \mathbf{a}$

6 a



$\overrightarrow{AM} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CM}$

$\overrightarrow{CM} = -\mathbf{a}$

$\overrightarrow{AM} = \mathbf{a} + \mathbf{b} - \mathbf{a}$

$= \mathbf{b}$

b $\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD}$

$= \mathbf{b} - 3\mathbf{a}$

c $\overrightarrow{MB} = \overrightarrow{MC} + \overrightarrow{CB}$

$= \mathbf{a} - \mathbf{b}$

d $\overrightarrow{DA} = \overrightarrow{DC} + \overrightarrow{CB} + \overrightarrow{BA}$

$= 3\mathbf{a} - \mathbf{b} - \mathbf{a}$

$= 2\mathbf{a} - \mathbf{b}$

7 a $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$= \mathbf{a} + \mathbf{b}$

b $\overrightarrow{OP} = \frac{5}{8}\left(\overrightarrow{OA} + \overrightarrow{AB}\right)$

$= \frac{5}{8}(\mathbf{a} + \mathbf{b})$

c $\overrightarrow{AP} = \overrightarrow{AB} + \overrightarrow{BP}$

$= \mathbf{b} - \frac{3}{8}(\mathbf{a} + \mathbf{b})$

$= \frac{5}{8}\mathbf{b} - \frac{3}{8}\mathbf{a}$

8 a $2\mathbf{a} - 6\mathbf{b} = 2(\mathbf{a} - 3\mathbf{b})$
Yes, parallel to $\mathbf{a} - 3\mathbf{b}$.

b $4\mathbf{a} - 12\mathbf{b} = 4(\mathbf{a} - 3\mathbf{b})$
Yes, parallel to $\mathbf{a} - 3\mathbf{b}$.

8 c $\mathbf{a} + 3\mathbf{b}$ is not parallel to $\mathbf{a} - 3\mathbf{b}$

d $3\mathbf{b} - \mathbf{a} = -1\ (\mathbf{a} - 3\mathbf{b})$
Yes, parallel to $\mathbf{a} - 3\mathbf{b}$.

e $9\mathbf{b} - 3\mathbf{a} = -3\ (\mathbf{a} - 3\mathbf{b})$
Yes, parallel to $\mathbf{a} - 3\mathbf{b}$.

f $\frac{1}{2}\mathbf{a} - \frac{2}{3}\mathbf{b} = \frac{1}{2}\left(\mathbf{a} - \frac{4}{3}\mathbf{b}\right)$
No, not parallel to $\mathbf{a} - 3\mathbf{b}$.

9 a i $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$= -\mathbf{a} + \mathbf{b}$

$= \mathbf{b} - \mathbf{a}$

ii $\overrightarrow{AP} = \frac{1}{2}\overrightarrow{AB}$

$= \frac{1}{2}\mathbf{a}$

iii $\overrightarrow{AQ} = \frac{1}{2}\overrightarrow{AC}$

$= \frac{1}{2}\mathbf{b}$

iv $\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AQ}$

$= -\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

$= \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a}$

b $\overrightarrow{PQ} = \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a}$

$\overrightarrow{BC} = \mathbf{b} - \mathbf{a}$

$\mathbf{b} - \mathbf{a} = 2\left(\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}\right)$

Therefore, the vectors $\overrightarrow{PQ}$ and $\overrightarrow{BC}$ are parallel.

10 a i $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -\mathbf{a} + \mathbf{a} + 2\mathbf{b}$

$= 2\mathbf{b}$

ii $\overrightarrow{CB} = \overrightarrow{CO} + \overrightarrow{OB}$

$= -3\mathbf{b} + \mathbf{a} + 2\mathbf{b}$

$= \mathbf{a} - \mathbf{b}$

b $\overrightarrow{AB} = 2\mathbf{b}$

$\overrightarrow{OC} = 3\mathbf{b}$

$2\mathbf{b} = \frac{2}{3}(3\mathbf{b})$

Therefore, the vectors $\overrightarrow{AB}$ and $\overrightarrow{OC}$ are parallel.

11 As the vectors are parallel

$5\mathbf{a} + 3\mathbf{b} = \frac{5}{2}(2\mathbf{a} + k\mathbf{b})$

$5\mathbf{a} + 3\mathbf{b} = 5\mathbf{a} + \frac{5k}{2}\mathbf{b}$

$3\mathbf{b} = \frac{5k}{2}\mathbf{b}$

$\frac{5k}{2} = 3$

$k = 1.2$