Variable acceleration 11B

1 a $s = 4t^4 - \frac{1}{t}$

i $v = \frac{ds}{dt} = 16t^3 + \frac{1}{t^2}$

ii $a = \frac{dv}{dt} = 48t^2 - \frac{2}{t^3}$

b $x = \frac{2}{3}t^3 + \frac{1}{t^2}$

i $v = \frac{dx}{dt} = 2t^2 - \frac{2}{t^3}$

ii $a = \frac{dv}{dt} = 4t + \frac{6}{t^4}$

c $s = (3t^2 - 1)(2t + 5)$
$= 6t^3 + 15t^2 - 2t - 5$

i $v = \frac{ds}{dt} = 18t^2 + 30t - 2$

ii $a = \frac{dv}{dt} = 36t + 30$

d $x = \frac{3t^4 - 2t^3 + 5}{2t} = \frac{3t^3}{2} - t^2 + \frac{5}{2t}$

i $v = \frac{dx}{dt} = \frac{9t^2}{2} - 2t - \frac{5}{2t^2}$

ii $a = \frac{dv}{dt} = 9t - 2 + \frac{5}{t^3}$

2 a $x = 2t^3 - 8t$

$v = \frac{dx}{dt} = 6t^2 - 8$

When $t = 3$, $v = 6 \times 3^2 - 8 = 46$

The velocity of the particle when $t = 3$ is 46 m s^{-1}.

b $a = \frac{dv}{dt} = 12t$

When $t = 2$, $a = 12 \times 2 = 24$

The acceleration of the particle when $t = 2$ is 24 m s^{-2}.

3 P is at rest when $v = 0$.

$12 - t - t^2 = 0$

$(4 + t)(3 - t) = 0$

$t = -4$ or $t = 3$

$t \geq 0$, so $t = 3$

$a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = -1 - 2t$

When $t = 3$, $a = -1 - 2 \times 3 = -7$

The acceleration of P when P is instantaneously at rest is $-7\ \mathrm{m\,s^{-2}}$, or $7\ \mathrm{m\,s^{-2}}$ in the direction of x decreasing.

4 $x = 4t^3 - 39t^2 + 120t$

$v = \dfrac{\mathrm{d}x}{\mathrm{d}t} = 12t^2 - 78t + 120$

P is at rest when $v = 0$.

$12t^2 - 78t + 120 = 0$

$2t^2 - 13t + 20 = 0$

$(2t - 5)(t - 4) = 0$

P is at rest when $t = 2.5$ and $t = 4$.

When $t = 2.5$, $x = 4(2.5)^3 - 39(2.5)^2 + 120(2.5) = 118.75$

When $t = 4$, $x = 4(4)^3 - 39(4)^2 + 120(4) = 112$

The distance between the two points where P is instantaneously at rest is $118.75 - 112 = 6.75$ m.

5 $v = kt - 3t^2$

a $a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = k - 6t$

When $t = 0$, $a = 4$

$k - 6 \times 0 = 4$

$k = 4$

b P is at rest when $v = 0$.

$4t - 3t^2 = 0$

$t(4 - 3t) = 0$

P is at rest when $t = 0$ and $t = \frac{4}{3}$.

When $t = \frac{4}{3}$, $a = 4 - 6 \times \frac{4}{3} = 4 - 8 = -4$

When P is next at rest, the acceleration is $-4\ \mathrm{m\,s^{-2}}$.

6 $s = \frac{1}{4}(4t^3 - 15t^2 + 12t + 30)$

$v = \dfrac{\mathrm{d}s}{\mathrm{d}t} = \frac{1}{4}(12t^2 - 30t + 12)$

6 The print head is at rest when $v = 0$.

$\frac{1}{4}(12t^2 - 30t + 12) = 0$

$12t^2 - 30t + 12 = 0$

$2t^2 - 5t + 2 = 0$

$(2t - 1)(t - 2) = 0$

The print head is at rest when $t = 0.5$ and $t = 2$.

When $t = 0.5$,

$s = \frac{1}{4}(4(0.5)^3 - 15(0.5)^2 + 12(0.5) + 30)$

$= \frac{1}{4}(0.5 - 3.75 + 6 + 30)$

$= 8.1875$

When $t = 2$,

$s = \frac{1}{4}(4(2)^3 - 15(2)^2 + 12(2) + 30)$

$= \frac{1}{4}(32 - 60 + 24 + 30)$

$= 6.5$

Distance between these two points $= 8.1875 - 6.5$

$= 1.6875$ cm

$= 1.7$ cm (1 d.p.)

The distance between the points when the print head is instantaneously at rest is 1.7 cm.